

2.4 Properties of Logarithms

OBJ: condense, expand, and solve logarithmic equations by using the properties of logarithms.

Since logarithms are inverses of _____ the properties of logarithms can be derived from the properties of exponents.

- When **expanding** logs, we want to have multiple logs being added or subtracted from each other.
- When **condensing** logs, we want to end with one log and multiple variables.

PRODUCT PROPERTY:

$$\log_b mn = \log_b m + \log_b n$$

The logarithm of a product (multiplication) is the same as the _____ of the logs

Example 1: Expand $\log_3(4)(7)$

You Try! Expand $\log xyz =$

Example 2: Condense $\log_6 10 + \log_6 x + \log_6 7$

You Try! Condense: $\log_4 x + \log_4 m + \log_4 9$

QUOTIENT PROPERTY:

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

The logarithm of a quotient (division) is the same as the _____ of the logs

Example 3: Expand: $\log_4 \frac{x}{y} =$

You Try! Expand $\log_7 \frac{5}{9} =$

Example 4: Condense $\log_4 10 - \log_4 x$

You Try! Condense $\log_2 x - \log_2 m$

EXPONENT PROPERTY:

$$\log_b m^p = p \log_b m$$

The logarithm of the power is the same as the _____ of the logs

Example 5: Expand $\log_5 x^4 =$

You Try! Expand $\log_{47} y^{\frac{1}{3}} =$

Example 6: Condense $5 \log_{12} x =$

You Try! Condense $\frac{7}{4} \log_9 z =$

Square Roots: If you ever see a square root in a problem, you must convert it to a rational exponent. *Remember – the index becomes the denominator of the exponent!

Example 7: Expand $\log_9 \sqrt{8z} =$

You Try! Expand $\log_{12} \sqrt[4]{\frac{x}{z}} =$

Putting it all together: Expand or condense the following logarithms using properties listed above.

a) Expand $\log_3 x^5 y^7 =$

b) Expand $\log_5 \frac{a^3}{b^7} =$

c) Expand $\log_5 \frac{g^6 h^2}{k^5} =$

d) Condense $5\log_2 x + 7\log_2 y$

e) Condense $6\log_5 g - 9\log_5 b$

f) Condense
 $7\log_4 x + \log_4 y - 6\log_4 z$

RULE: If there are multiple logs on the same side of an equation, _____ before solving, then apply the rules from Day 1 (cancel, swoosh, or COB)

Example 8: $\log_3 5 - \log_3 x = \log_3 10$

Example 9: $4\log_2 x + \log_2 5 = \log_2 405$

1. Expand $\log_3 \sqrt{\frac{x^5 y^6}{z^7}} =$

2. Condense $4\log_2 c + 8\log_2 d - \log_2 e$

3. Solve for x: $3\log_5 x - \log_5 4 = \log_5 16$

4. Solve for x: $\log a = \log(4a - 9)$

5. Solve for x: $\log(-3m - 1) = \log(-4m - 6)$

6. Solve for x: $5\log_{19} 2 - \log_{19} x = \log_{19} 8$