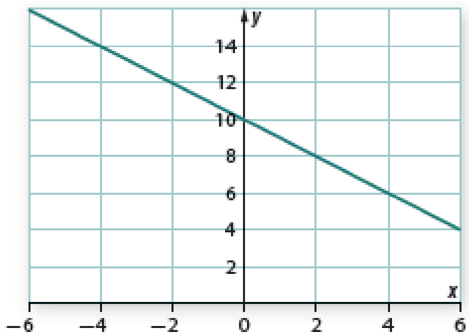
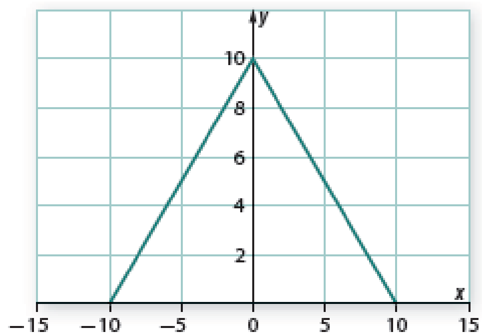
8.1.2 Practice

6 Which of these graphs represent functions that have inverses? Be prepared to justify each answer.

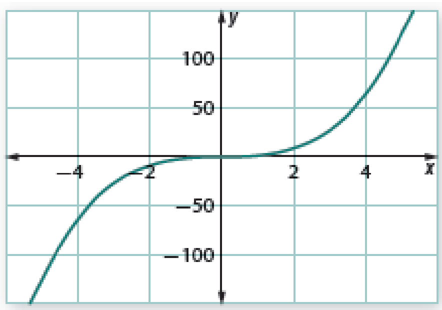
Graph I $y = 10 - x$



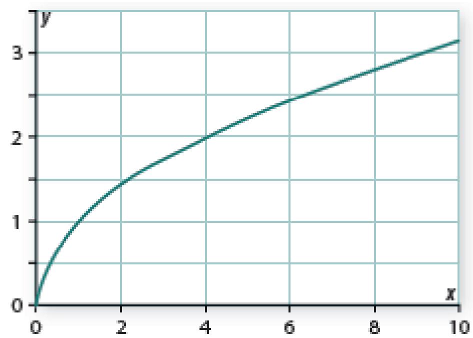
Graph II $y = 10 - |x|$



Graph III $y = x^3$



Graph IV $y = \sqrt{x}$



7 Find rules for the inverses of the following functions.

a. $f(x) = 4x - 5$

b. $g(x) = 8x^2$ (domain $x \geq 0$)

c. $h(x) = \frac{5}{x}$ (domain $x \neq 0$)

d. $k(x) = -5x + 7$

8 The following statements describe situations where functions relate two quantitative variables. For each situation:

- if possible, give a rule for the function that is described.
- determine whether the given function has an inverse.
- if an inverse exists, give a rule for that inverse function (if possible) and explain what it tells about the variables of the situation.

a. If regular gasoline is selling for \$3.95 per gallon, the price of any particular purchase p is a function of the number of gallons of gasoline g in that purchase.

b. If a school assigns 20 students to each mathematics class, the number of mathematics classes M is a function of the number of mathematics students s in that school.

c. The area of a square A is a function of the length of each side s.

d. The number of hours of daylight d at any spot on Earth is a function of the time of year t.

11 Consider the geometric transformation with coordinate rule $(x, y) \rightarrow (x + 3, y + 2)$.

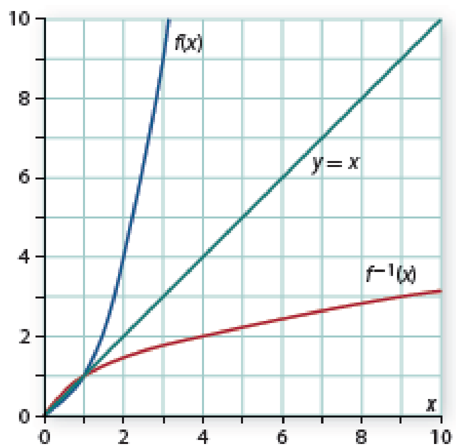
a. What kind of transformation is defined by that rule?

b. What is the rule for the inverse of that transformation?

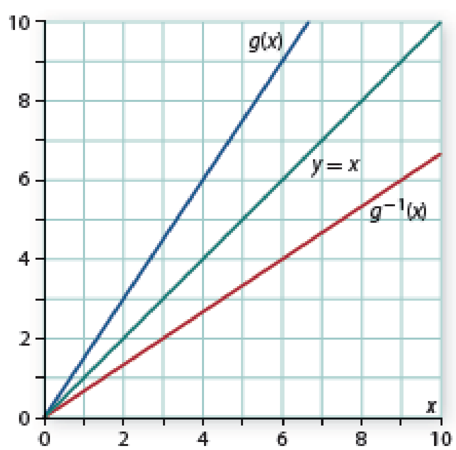
14 The following graphs show pairs of functions that are inverses of each other and the line $y = x$. For each pair of functions:

- find two points (a, b) and (c, d) on one graph and show that the points (b, a) and (d, c) are on the other graph.
- explain why the transformation $(x, y) \rightarrow (y, x)$ maps every point of one graph onto a point of the other graph.

a. $f(x) = x^2$ and $f^{-1}(x) = \sqrt{x}, x \geq 0$



b. $g(x) = 1.5x$ and $g^{-1}(x) = \frac{2}{3}x$



16 Rounding is a function used often in calculator or computer work with numeric data.

a. If $r_2(x)$ rounds every number to two decimal places, find these results from using that function.

i. $r_2(3.141)$ **ii.** $r_2(2.718)$ **iii.** $r_2(2.435)$

b. Explain why it is or is not possible to find the value of x when you know the value of $r_2(x)$.

19 When Brianna looked up "inverse function" on the Internet, she found a sentence that stated, "A function has an inverse if and only if it is a one-to-one function." She wondered what the phrase "one-to-one" means.

> Does it mean that for each value of x
> there is exactly one paired value of y?
> Or does it mean that for each value of y
> there is exactly one paired value of x?

Based on your investigation of inverse functions, what do you think the one-to-one condition tells about a function?

20 The word "inverse" is used in several different ways in algebra. For example, we say that -7 is the *additive inverse* of 7 because $-7 + 7 = 0$. Similarly, we say that $\frac{7}{2}$ is the *multiplicative inverse* of $\frac{2}{7}$ because $\left(\frac{7}{2}\right)\left(\frac{2}{7}\right) = 1$.
How is the use of the word "inverse" in the phrase "inverse function" similar to its use in the phrases "additive inverse" and "multiplicative inverse"?

22 Here are two ways to think about finding the rule for an inverse function.

Brandon's Strategy: If I know the rule for $f(x)$ as an equation relating y and x, I simply swap the symbols y and x and then solve the resulting equation for y. For example, if $f(x) = 3x + 5$ or $y = 3x + 5$, then I swap y and x to get $x = 3y + 5$ and solve for y to get the rule $y = \frac{x-5}{3}$ for the inverse.

Luisa's Strategy: If I know the rule for $f(x)$ as an equation relating y and x, I solve that equation for x in terms of y and then use that equation to write the rule for the inverse of f. For example, if $f(x) = 3x + 5$ or $y = 3x + 5$, I solve for x to get $x = \frac{y-5}{3}$, that tells me the inverse function has rule $f^{-1}(x) = \frac{x-5}{3}$ or $y = \frac{x-5}{3}$.

a. Will either or both of these strategies always give the correct inverse function rule?

b. Which of the two strategies (or some other strategy of your own) makes most sense to you as a way of thinking about inverse functions and their rules?

35 Write these expressions in equivalent form as products of linear factors.

a. $x^2 + 7x$
b. $x^2 + 7x + 12$
c. $x^2 + 7x - 8$
d. $x^2 - 49$
e. $x^2 - 6x + 9$
f. $3x^2 - 2x - 8$