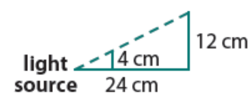
3.1.1 Solutions

1 a. The scale factor from the preimage triangle to the image triangle is $\frac{10}{4} = \frac{5}{2}$, so the lengths of the images of the 3-cm and 5-cm sides are 7.5 cm and 12.5 cm, respectively.

b. In this case, the scale factor from the preimage triangle to the image triangle is $\frac{3}{2}$, so the lengths of the images of the 3-cm and 4-cm sides are 4.5 cm and 6 cm, respectively.

c. Since you want the 4-cm side to increase to 12 cm, the needed scale factor from the preimage triangle to the image triangle is 3. Since the flashlight is 24 cm from the wall, placing the cardboard triangle 8 cm from the flashlight will produce the scale factor of 3 $\left(\frac{1}{3}\text{ of 24 is 8}\right)$.



2 a. i. Photo 2: Both photos were taken parallel to the plane of the footwear impression, yet Photo 2 provides more useful information regarding the actual size of the impression because a known measurable object (standard dollar bill) is positioned next to the impression. It is impossible to determine anything about the actual size of the print in Photo 1 because there are no known measurable objects in the photo.

ii. Photo 3: Both photos include the dollar bill in order to determine the actual size of the print, yet the scaled image shown in Photo 4 provides less reliable information because the camera was not aligned parallel to the plane of the object (the image is skewed).

b. Since the actual size of a dollar bill is known (6.14 in. × 2.61 in. ≈ 156 mm × 66 mm), a technician could use the dimensions of the dollar bill in Photo 2 or Photo 3 to determine a scale factor from the dimensions of the dollar in the photo to its actual dimensions. With this scale factor, the actual dimensions of the footprint can be determined.

Approximate dimensions of dollar bill from Photo 2: 22 mm × 9 mm

$(\textit{actual dimensions}) = k(\textit{photo dimensions})$, or $k = \frac{\textit{actual dimensions}}{\textit{photo dimensions}}$

$\frac{156\text{ mm}}{22\text{ mm}} \approx 7.09$ and $\frac{66\text{ mm}}{9\text{ mm}} \approx 7.33$

A reasonable estimate for a scale factor is 7.2.

Approximate dimensions of impression at longest and widest parts from Photo 2: 44 mm × 17 mm

$\textit{actual length} \approx (7.2)(44\text{ mm}) = 317\text{ mm}$

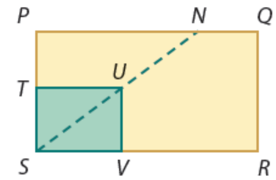
$\textit{actual width} \approx (7.2)(17\text{ mm}) = 122\text{ mm}$

12 a. $\frac{XY}{AB} = \frac{YZ}{BC} = \frac{XZ}{AC}$ since each ratio is equal to k, the scale factor.

b. Each ratio in this proportion is the reciprocal of one in Part a; thus, all equal $\frac{1}{k}$.

c. Two polygons are similar provided their corresponding angles have the same measure and corresponding sides are proportional.

18 **a.** $m\angle A = m\angle E$ (right angles) and $m\angle ADB = m\angle EDF$ (common angles). So, $\triangle ABD \sim \triangle EFD$ (AA Similarity Theorem). Thus, corresponding sides $\overline{AD}$ and $\overline{ED}$, and $\overline{AB}$ and $\overline{EF}$, are related by the same scale factor (corresponding sides of similar triangles). Since opposite sides of a rectangle are congruent, $\overline{CD}$ and $\overline{GD}$, and $\overline{BC}$ and $\overline{FG}$ are also related by the same scale factor as the other corresponding sides. Thus, the two rectangles are similar (definition of similar polygons).



Using similar reasoning, $\triangle PSN \sim \triangle TSU$. So, $\frac{PN}{TU} = \frac{PS}{TS} = k$. Since $PQ > PN$, $\frac{PQ}{TU} \neq k$.

b. Yes, since opposite sides are parallel, the corresponding angles ($\angle A$ and $\angle E$) are the same measure, although not right angles. The remainder of the reasoning in Part a does not rely on the polygons being rectangles.

23 **a.** **i.** The pizzas are similar because the scale factor from the smaller pizza to the larger pizza is 1.2. The lengths of the sides of the rectangle are proportional. (SSS Similarity Theorem)

ii. 1.2

iii. The dimensions of the new pizza are $(1.2)(20) = 24$ inches and $(1.2)(15) = 18$ inches.

iv. $\textit{perimeter of original pizza} = 20(2) + 15(2) = 70$ inches
$\textit{perimeter of new pizza} = 24(2) + 18(2) = 84$ inches
$\frac{\textit{perimeter of new pizza}}{\textit{perimeter of original pizza}} = \frac{84}{70} = 1.2$, which is equal to the scale factor.

v. $\dfrac{\textit{area of new pizza}}{\textit{area of original pizza}} = \dfrac{24(18)}{15(20)} = 1.44 = (1.2)^2$, which is equal to the scale factor.

b. **i.** $(\textit{original area})1.2 = \textit{new area}$, so $(20)(15)(1.2) = 360$. An alternative solution method is to calculate 20% of the original area, and add it to the original area: $(20)(15)(0.2) = 60$, so the new area is $300 + 60 = 360$ square inches.

ii. Since *original pizza* $\sim$ *enlarged pizza*, notice that $\dfrac{\textit{area of new pizza}}{\textit{area of original pizza}} = k^2$, so $k = \sqrt{\dfrac{360}{300}} = \sqrt{1.2}$. The new dimensions are $\sqrt{1.2} \cdot 20 \approx 21.9$ inches and $\sqrt{1.2} \cdot 15 \approx 16.4$ inches.

iii. The scale factor relating the original pizza to the new pizza is $\sqrt{1.2} \approx 1.0954$.

c. The interpretation in Part a of a 20% increase in the lengths of the edges of the pizza favors the consumer because the total area in that case is $(18)(24) = 432$ square inches and in Part b is 360 square inches.

31 **a.** If $a^2 > 0$, then $a > 0$. This converse is *false*; $(-2)^2 = 4 > 0$, but $-2 < 0$.

b. If ab is a multiple of 4, then a is an even number and b is an even number.
This converse is *false*. If $a = 3$ and $b = 8$, then $3(8) = 24$ is a multiple of 4, but $a = 3$ is not an even number.

c. All quadrilaterals that have four right angles are rectangles.
True: A quadrilateral with four right angles is defined to be a rectangle.

d. If the correlation for two variables is positive, then the linear regression line has positive slope.
True: The correlation and slope of the linear regression line always have the same sign.