Solutions 3.1.2

4 **a.** $\triangle ABC \sim \triangle EDF$:
$m\angle D = 180° - 87° - 61° = 32° = m\angle B$ and $m\angle F = 61° = m\angle C$.
So by the AA Similarity Theorem, the triangles are similar.

b. $\triangle MNO \sim \triangle RTS$:
$m\angle N = 180° - 90° - 42° = 48° = m\angle T$ and $m\angle M = 90° = m\angle R$.
So by the AA Similarity Theorem, the triangles are similar.

c. $\triangle UVW \sim \triangle YZX$ and $\triangle UVW \sim \triangle XZY$:
$\triangle UVW$ and $\triangle YZX$ are isosceles triangles, so $m\angle U = m\angle W = 70°$ and $m\angle X = m\angle Y = \frac{180° - 40°}{2} = 70°$. So by the AA Similarity Theorem or the SAS Similarity Theorem, the triangles are similar.

d. $\triangle FGE \sim \triangle HGI$:
Since $\angle FGE$ and $\angle HGI$ are vertical angles, they have the same measure. Since $\angle F$ and $\angle H$ are right angles, they have the same measure. So by the AA Similarity Theorem, the triangles are similar.

e. $\triangle MNO \sim \triangle QPO$:
$PO = 2NO$, $QO = 2MO$, and $m\angle NOM = m\angle POQ$ (vertical angles). So by the SAS Similarity Theorem, the triangles are similar.

f. $\triangle ABC$ is not similar to $\triangle PQR$ or $\triangle PRQ$ because the corresponding sides are not related by the same scale factor.

5 **a.** $\triangle ABC \sim \triangle ADB$, $\triangle ABC \sim \triangle BDC$, $\triangle BDC \sim \triangle ADB$

b. Student strategies will likely utilize the AA Similarity Theorem.

(1) $\triangle ABC \sim \triangle ADB$; use the right angles and the common angle at A.

(2) $\triangle ABC \sim \triangle BDC$; use the right angles and the common angle at C.

(3) $\triangle BCD \sim \triangle ABD$; use the right angles and identify another pair of angles having the same measure based on corresponding pairs of angles from (1) or (2) above. For example, $m\angle ABD = m\angle ACB$ from (1), so $m\angle ABD = m\angle BCD$ (C is the common angle).

14 **a.** The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments into which the altitude divides the hypotenuse.

b. $\frac{4}{x} = \frac{x}{9}$, so $x = 6$.

$\frac{7}{x} = \frac{x}{12}$, so $x = \sqrt{84}$.

c. $\frac{a+b}{2} \geq \sqrt{ab}$

d. **i.** RT

Since T is the center, $\overline{RT}$ is a radius. So, $RT = \frac{a+b}{2}$, the arithmetic mean of a and b.

QU

Assuming from the diagram that point Q is on the circle, $\angle PQS$ is a right angle (inscribed in a semicircle). $\angle PUQ$ is a right angle, so $\overline{QU}$ is an altitude of $\triangle PQS$. By Part a, QU is the geometric mean $\sqrt{ab}$.

Alternatively, some students may use the Pythagorean Theorem and algebraic reasoning to deduce that $QU = \sqrt{ab}$ as follows.

$\overline{QU}$ is a leg of $\triangle PUQ$ and $\triangle QUS$.

So, $QU^2 = QS^2 - b^2$ and $QU^2 = PQ^2 - a^2$.

$2QU^2 = QS^2 + PQ^2 - (a^2 + b^2)$ and $QS^2 + PQ^2 = (a+b)^2$

So, $2QU^2 = (a+b)^2 - (a^2 + b^2) = 2ab$.

$QU^2 = ab$

Thus, $QU = \sqrt{ab}$, the geometric mean.

ii. RT (the arithmetic mean) is the length of a radius of the circle. QU (the geometric mean) is either equal to RT or less than RT since the perpendicular distance from the diameter $\overline{PS}$ to any point Q on the circle is less than or equal to RT, the radius of the circle.

e. The geometric mean and the arithmetic mean are equal if and only if $a = b$.

26 Since $(a+b)^2$ and $(a-b)^2$ have terms of $2ab$ and $-2ab$, we could start by either adding or subtracting these binomials. But adding them, once the trinomials are found, will lose the ab term. So instead, subtract them:

$(a+b)^2 - (a-b)^2 = 4ab$

$(a-b)^2 \geq 0$, so $(a+b)^2 \geq 4ab$.

$\frac{(a+b)^2}{4} \geq ab$ and thus, since $a > 0$ and $b > 0$, $\frac{a+b}{4} \geq \sqrt{ab}$.

34 **a.** $9t = 72$

$t = 8$

b. $5(m+4) = 12m$

$5m + 20 = 12m$

$20 = 7m$

$\frac{20}{7} = m$

c. $3(y-3) = 10(2y+5)$

$3y - 9 = 20y + 50$

$-17y = 59$

$y = -\frac{59}{17}$

36 **a.** $x^2 - 25$

b. $-6x^2 + 33x + 18$

c. $-3x^2 + 13x + 3$

d. $100x^2 - 120x + 36$