Solutions 6.2.2

5 The height of the rotating capsule is $75 \sin(270° + \theta) + 75$, or $75 \sin(\theta - 90°) + 75$. The answers below are rounded to the nearest tenth of a foot.

a. 4.5

b. 22.0

c. 68.5

d. 75

e. 112.5

f. 150

g. 75

h. 37.5

i. 4.5

6 **a.** (23, 19.3)

b. (−18.8, 6.8)

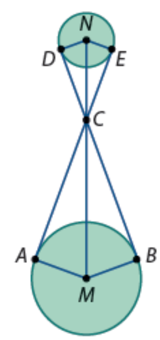
c. (−60.6, −35)

d. (−9.6, −11.5)

e. (0, −45)

f. (25.7, −30.6)

28 **a.** See the figure on the right. Angles A, B, D, and E are right angles since they are formed by a radius and a tangent line at the point of tangency. Note that $\triangle ACM \cong \triangle BCM$ and $\triangle DCN \cong \triangle ECN$ (SSS Congruence Theorem). $\angle ACM \cong \angle ECN$ since they are vertical angles. So, $\triangle ACM \sim \triangle ECN$ (AA Similarity Theorem). Thus, the four triangles are similar to one another. Some pairs are similar with scale factor of 1.



b. $\overline{AE}$ is one of the two common tangents. Let $CM = x$. Then $CN = 30 - x$. By similar triangles, $\frac{AM}{EN} = \frac{CM}{CN}$. Substituting, $\frac{8}{4} = \frac{x}{30-x}$. Therefore, $240 - 8x = 4x$, so $12x = 240$ and $CM = x = 20$ cm. It follows that $CN = 30 - 20 = 10$ cm. Now apply the Pythagorean Theorem in $\triangle ACM$ and $\triangle ECN$.

$$AC^2 = CM^2 - AM^2$$
$$AC = \sqrt{20^2 - 8^2} = \sqrt{336} \approx 18.3 \text{ cm}$$

By similar triangles, $\frac{DN}{AM} = \frac{CE}{AC}$, so $\frac{4}{8} = \frac{CE}{18.3}$; $CE = 9.2$ cm. Therefore, the length of common tangent $\overline{AE}$ is $AE = AC + CE \approx 27.5$ cm.

c. The belt's length is the sum of the two common tangents, or 55 cm, plus the portion of the circumference of each circle that the belt is touching. Determine the latter by first finding the common measure of central angles AMB and DNE using trigonometry.

$$m\angle AMC = m\angle BMC = \cos^{-1}\frac{8}{20} \approx 66.4°$$

Therefore, $m\angle AMB \approx 132.8°$. Also by similar triangles, $m\angle DNE \approx 132.8°$. The length of the belt touching the larger circle is:

$$\frac{360 - 132.8}{360} \cdot (2\pi \cdot 8) \approx 31.7 \text{ cm.}$$

It follows by similar triangles that the length of the belt touching the smaller circle is half of this result or about 15.9 cm. The total length of the belt is:

$$31.7 + 15.9 + 2(27.5) = 102.6 \text{ cm.}$$

37 **a.** $r = \frac{C}{2\pi}$

b. $b_2 = \frac{2A}{h} - b_1$

c. $c = \pm\sqrt{\frac{E}{m}}$

d. $v = \frac{m}{d}$

39 **a.** $\ell \parallel m$

b. Not enough

c. $a \parallel b$

d. $\ell \parallel m$; $a \parallel b$