6 **a–d.** The graphs of $y = 10 - x$, $y = x^3$, and $y = \sqrt{x}$ have inverses because each domain value has its own private range value. No distinct domain values have the same range value.

7 **a.** $f^{-1}(x) = \frac{x + 5}{4}$, or $f^{-1}(x) = 0.25x + 1.25$

b. $g^{-1}(x) = \sqrt{\frac{x}{8}}$

c. $h^{-1}(x) = \frac{5}{x}$

d. $k^{-1}(x) = \frac{x - 7}{-5}$, or $k^{-1}(x) = -0.2x + 1.4$

8 **a.** $p = 3.95g$; This function has an inverse. Ignoring the inevitable rounding of prices to nearest penny, the price function does have an inverse. If the total cost of a purchase is known, then the number of gallons in the purchase can be determined by $g = \frac{p}{3.95}$.

b. $M = \frac{s}{20}$. This function has an inverse. If the school manages to exactly meet the prescribed student-to-math-class ratio, then it will be possible to calculate the number of mathematics students in the school from the number of teachers by $s = 20M$.

c. $A = s^2$. This function has an inverse. If one knows the area of a square, it is always possible to calculate the length of each side by $s = \sqrt{A}$. The domain is restricted to $s > 0$.

d. The given information does not really allow construction of a function rule because it depends on the latitude of the particular spot. An inverse for this function does not exist. The number of hours of daylight at any spot on Earth is a periodic function for which times that are at symmetric points on either side of the summer or winter solstice will have the same number of hours of daylight.

Thus, the function that gives hours of daylight at any time of the year cannot be inverted to find time of year from hours of daylight. In general, there will always be two times of the year that have each possible number of daylight hours.

11 **a.** The given rule defines a translation that maps every point of the plane to a point that is 3 units to the right and up 2 units.

b. $(x, y) \rightarrow (x - 3, y - 2)$

14 **a.** For example, the points (0, 0) and (2, 4) are on the graph of $f(x)$ and (0, 0) and (4, 2) are on the graph of $f^{-1}(x)$.

Proof:

If (x, x^2) is on the graph of $f(x) = x^2$, then $f^{-1}(x^2) = \sqrt{x^2} = x$, so $(x^2, \sqrt{x^2}) = (x^2, x)$ is on the graph of $f^{-1}(x)$.

b. For example, the points (0, 0) and (4, 6) are on the graph of $g(x)$ and the points (0, 0) and (6, 4) are on the graph of $g^{-1}(x)$.

Proof:

If $(x, 1.5x)$ is on the graph of $g(x) = 1.5x$, then $(1.5x, x) = \left(1.5x, \frac{2}{3}(1.5x)\right)$ is on the graph of $g^{-1}(x)$.

16 **a.** **i.** 3.14 (This is a rounded value of π.)

ii. 2.72 (This is a rounded value of e, which will be introduced in Course 4.)

iii. 2.44 (This rounding uses the convention that for any number that is at least half-way to the next higher digit, you round up.)

b. There are many numbers that round to the same number. For example, 3.414 and 3.413 both round to 3.41. So after rounding has occurred, we cannot recover the original number.

19 The phrase "one-to-one" means that for each value of y, there is exactly one paired value of x. In other words, no y value is the image of two or more values of x. The other meaning that Arrillio was considering is actually a defining condition for a correspondence of values to be a function. That is, each x value in the domain is assigned exactly one image in the range. In the visual language of function arrow diagrams, a correspondence is not a function if there are two arrows starting at a domain point. A function is not one-to-one if there are two arrows ending at a range point.

20 The additive inverse of 7 is -7 because $7 + -7 = 0$. Essentially adding the inverse -7 has the effect of undoing the addition of 7. You can think of this property as retrieving the original number. For example, $(n + 7) + (-7) = n$. Similarly, multiplying by 7 and then by $\frac{1}{7}$ $\left(\text{because } 7 \times \frac{1}{7} = 1\right)$ has the effect of undoing the multiplication by 7 and retrieving the original number. That is $(n \times 7) \times \left(\frac{1}{7}\right) = n$.

22

a. The two strategies both work. In fact, the "swap x and y and then solve for y in terms of x" method seems to be the standard strategy.

b. Students will have their own preference of a strategy that makes the most sense to them.

35

a. $x(x + 7)$

b. $(x + 3)(x + 4)$

c. $(x + 8)(x - 1)$

d. $(x + 7)(x - 7)$

e. $(x - 3)(x - 3) = (x - 3)^2$

f. $(3x + 4)(x - 2)$