

SOLVING RATIONAL EQUATIONS

- Multiply the equation by the Least Common Denominator
- Simplify, Solve, Check answers.

$$1) \frac{5x}{x-2} + 7 = \frac{10}{x-2}$$

$$5x + 7(x-2) = 10$$

$$5x + 7x - 14 = 10$$

$$12x = 24$$

$$x = 2 \quad \boxed{\text{no sol.}}$$

$$3) \frac{3}{x^2+x} + \frac{3}{x+1} = -\frac{1}{x}$$

$$3 + 3x = -1(x+1)$$

$$3 + 3x = -x - 1$$

$$4 = -4x$$

$$\boxed{-1 = x}$$

$$5) \frac{x}{x-2} + \frac{6}{x^2+3x-10} = \frac{2}{x-2}$$

$$x(x+5) + 6 = 2(x+5)$$

$$x^2 + 5x + 6 = 2x + 10$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$\boxed{x = -4, x = 1}$$

Write an equation and solve it to find the solution to each problem.

- 7) John can mow a lawn in 4 hours. When Melissa helps him, they can mow the lawn in $2\frac{1}{2}$ hours. How long would it take Melissa to mow the lawn?

$$\frac{2.5}{4} + \frac{2.5}{x} = 1$$

$$2.5x + 10 = 4x$$

$$10 = 1.5x$$

$$\boxed{6\frac{2}{3} \text{ hours}}$$

$$2) \frac{3x}{x+1} = \frac{12}{x^2-1} + 2$$

$$3x(x-1) = 12 + 2x^2 - 2$$

$$3x^2 - 3x = 12 + 2x^2 - 2$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$\boxed{x = 5, x = -2}$$

$$4) \frac{1}{d+4} = \frac{2}{d^2+3d-4} + \frac{1}{d}$$

$$d-1 = 2 + d+4$$

$$d-1 = d+6$$

$$0 = 7 \quad \text{no sol.}$$

$$6) \frac{w}{w-1} + w = \frac{4w-3}{w-1}$$

$$w + w(w-1) = 4w-3$$

$$w + w^2 - w = 4w-3$$

$$w^2 - 4w + 3 = 0$$

$$(w-3)(w-1) = 0$$

$$\boxed{w = 3, w = 1}$$

- 8) A water tank is filled by pipes from 2 wells. The first pipe can fill the tank in 4 days. The second pipe can fill the tank in 6 days. How long will it take to fill the tank using both pipes?

$$\frac{x}{4} + \frac{x}{6} = 1$$

$$3x + 2x = 12$$

$$5x = 12$$

$$\boxed{x = 12\frac{2}{5} \text{ days}}$$

MULTIPLYING/DIVIDING RATIONALS

- Factor everything that you can--- Remember to KEEP-CHANGE-FLIP for division.
- Simplify by canceling out common factors

$$9) \frac{\cancel{x+3}}{x^2-4x+4} \cdot \frac{\cancel{(x-2)(x+1)}}{\cancel{(x-2)(x-2)} \cdot \cancel{(x+3)(x+1)}} = \boxed{\frac{1}{x-2}}$$

$$10) \frac{x^2-x-12}{3x+9} \div \frac{x^2+x-20}{x+5} = \frac{\cancel{(x-4)(x+3)}}{3\cancel{(x+3)}} \cdot \frac{\cancel{x+5}}{\cancel{(x+5)(x-4)}} = \boxed{\frac{1}{3}}$$

$$11) \frac{\cancel{(x-4)(x+5)}}{x^2-x-12} \times \frac{\cancel{2x-3}(x+6)}{2x^2+9x-18} = \frac{\cancel{(2x-3)(x-4)} \cdot 4(x+3)}{2x^2-11x+12}$$

$$\boxed{\frac{x+6}{4}}$$

$$12) \frac{5x+35}{9x^2-1} \div \frac{x^2+9x+14}{30x+10} = \frac{5\cancel{(x+7)}}{\cancel{(3x+1)}(3x-1)} \cdot \frac{10\cancel{(3x+1)}}{\cancel{(x+7)}(x+2)}$$

$$\boxed{\frac{50}{(3x-1)(x+2)}}$$

ADDING/SUBTRACTING RATIONALS

- Factor the denominators to get a common denominator
- COMBINE THE NUMERATORS TOGETHER after you have gotten the common denominator
- Factor your result IF POSSIBLE... then and only then, you can have a cancel party.

$$13) \frac{\cancel{(y+4)} \cdot 3}{\cancel{(y+4)}(y+3)} + \frac{2y}{\cancel{(y+3)(y+4)}} = \frac{3y+12+2y}{(y+3)(y+4)} = \boxed{\frac{5y+12}{(y+3)(y+4)}}$$

$$14) \frac{x}{x^2+2x-8} - \frac{2(x+1)}{x^2-2x} + \frac{5(x-2)}{x^2+4x} = \frac{x(x-2)(x+4) - 2(x+1)x(x-2) + 5(x-2)x(x+4)}{x(x-2)(x+4)}$$

$$= \boxed{\frac{x^2+3x-18}{x(x-2)(x+4)}}$$

$$15) \frac{\cancel{(x+1)(x+1)}}{x^2-7x+10} + \frac{3\cancel{(x-5)}}{\cancel{(x-2)(x+1)}} = \frac{x^2+2x+1+3x-15}{(x+1)(x-5)(x-2)}$$

$$= \frac{x^2+5x-14}{(x+1)(x-5)(x-2)} = \frac{\cancel{(x+7)}(x-2)}{\cancel{(x+1)(x-5)}(x-2)} = \boxed{\frac{x+7}{(x+1)(x-5)}}$$

$$16) \frac{5}{x-6} + \frac{1}{x} \cdot \frac{x}{x} = \frac{5}{x^2} + \frac{x}{x^2} = \frac{5+x}{x^2} \div \frac{x-6}{1} = \frac{5+x}{x^2} \cdot \frac{1}{x-6} = \boxed{\frac{(5+x)}{x^2(x-6)}}$$

RATIONAL FUNCTIONS GRAPHING

- FACTOR the Numerator and Denominator.
- Find any VERTICAL and HORIZONTAL asymptotes and HOLES that may exist.
- Use test points to sketch an accurate graph.

$$1. y = \frac{\cancel{(x-5)}(x+2)}{x^2 + 3x - 10}$$

$$\frac{x^2 - 25}{\cancel{(x-5)}(x+5)}$$

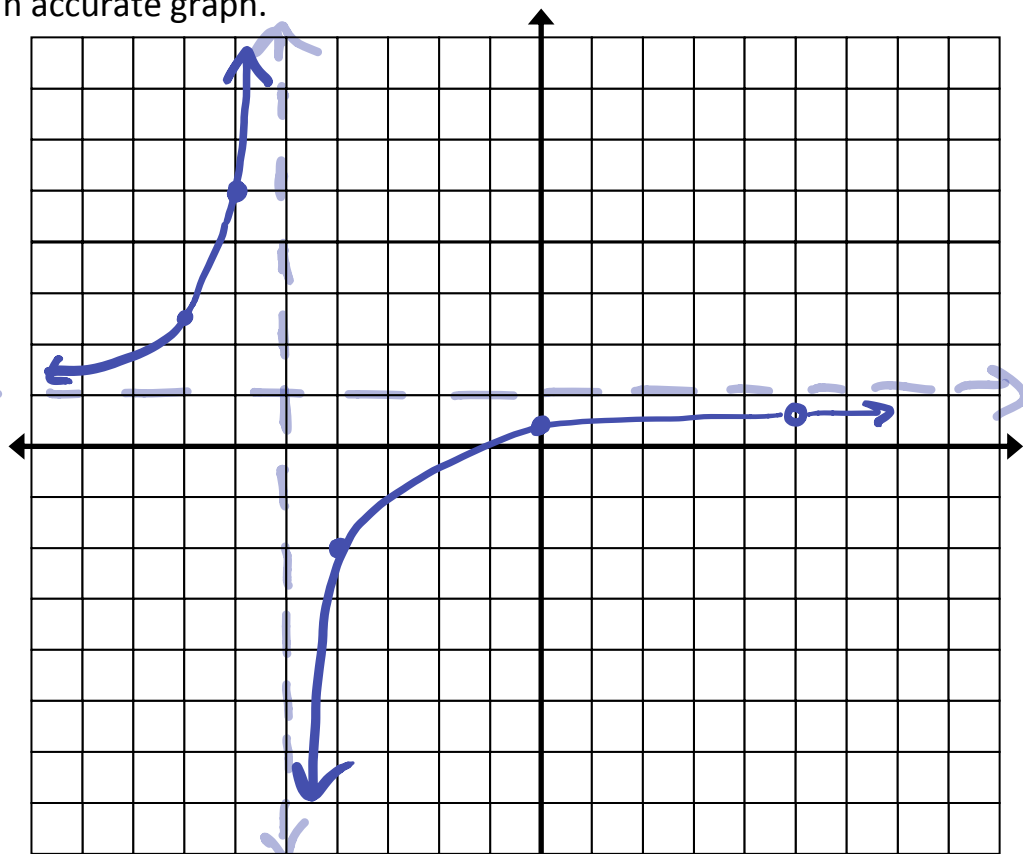
$$y = \frac{x+2}{x+5}$$

Asymptotes:

Vert: $x = -5$

HA: $y = 1$

Holes: $x = 5$



$$2. f(x) = \frac{2x \cancel{(x-3)}}{2x^2 - 6x}$$

$$\frac{x^2 + x - 12}{\cancel{(x-3)}(x+4)}$$

$$y = \frac{2x}{x+4}$$

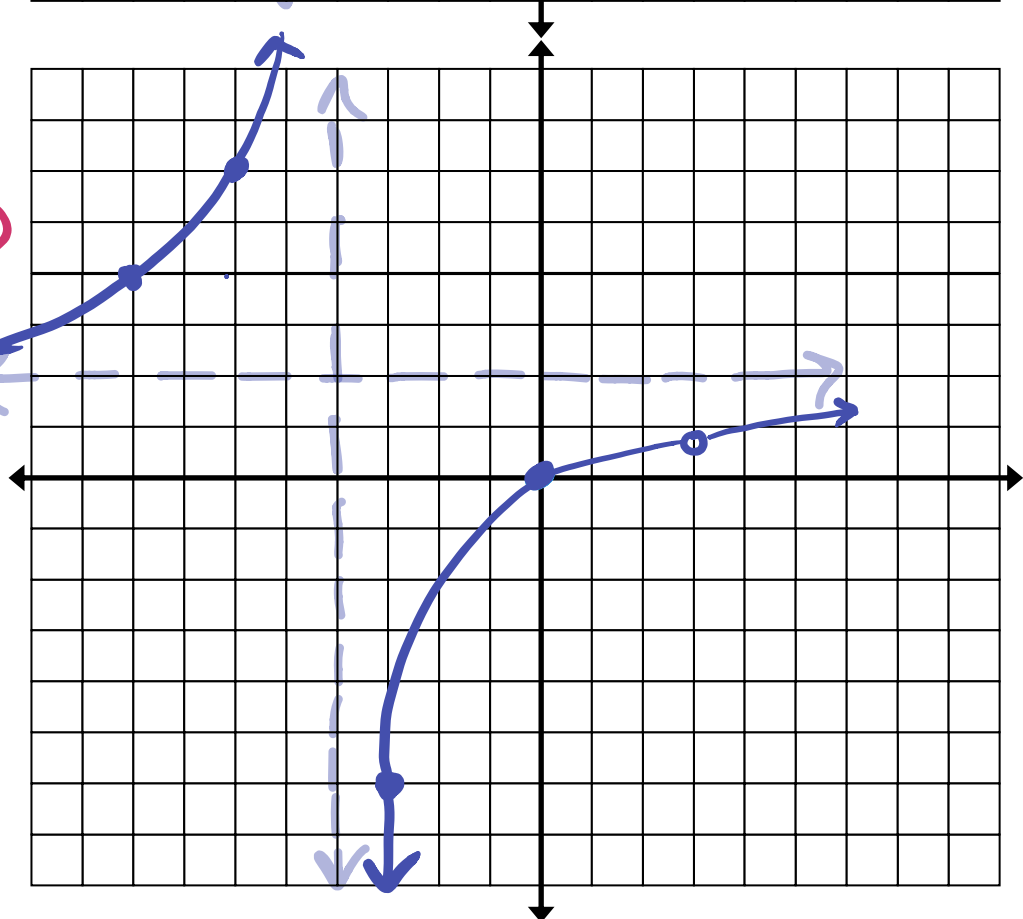
$\frac{-10}{-1}$
 $\frac{-16}{-4}$ $\frac{-12}{-2}$ $\frac{-6}{-1}$

Asymptotes:

Vert: $x = -4$

HA: $y = 2$

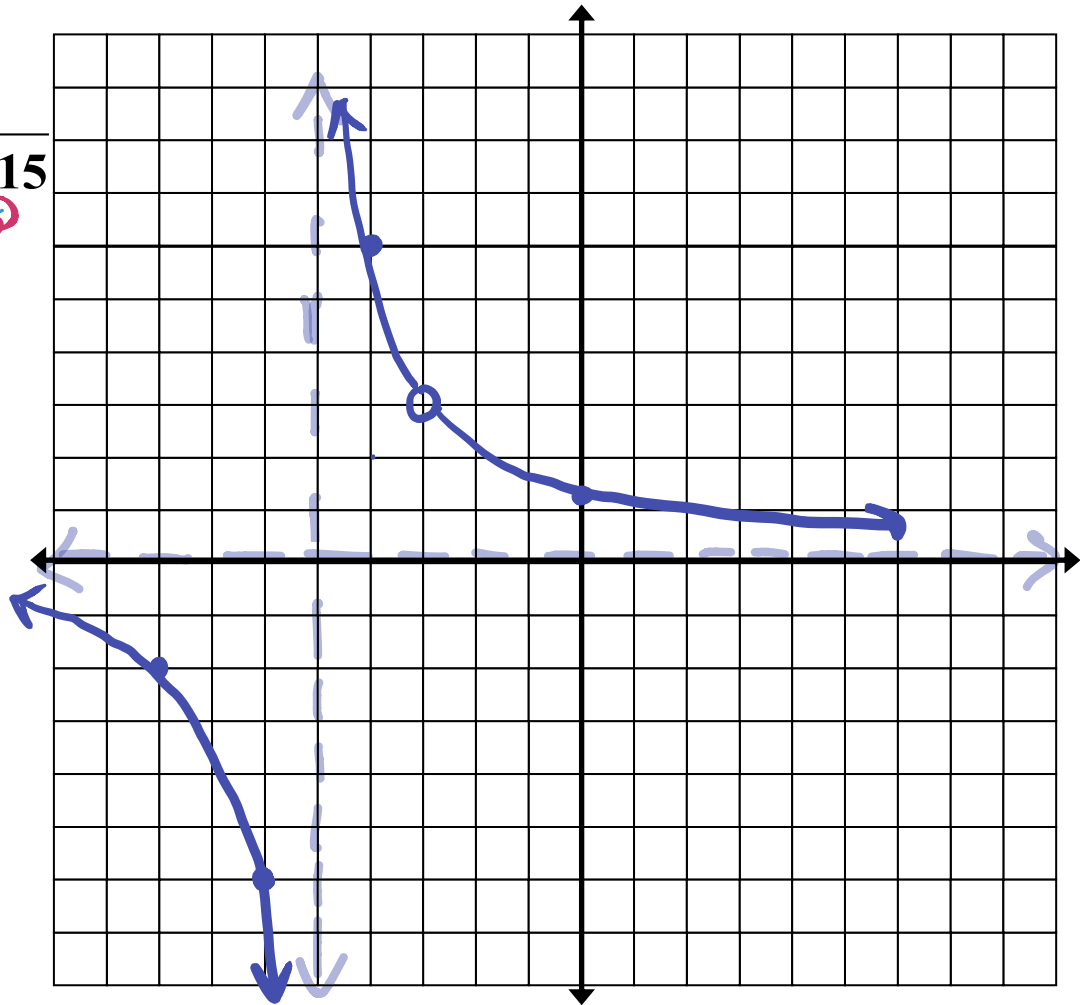
Holes: $x = 3$



$$3. g(x) = \frac{6x + 18}{x^2 + 8x + 15}$$

$\frac{6(x+3)}{(x+5)(x+3)}$

$$y = \frac{6}{x+5}$$



Asymptotes:

Vert: $x = -5$

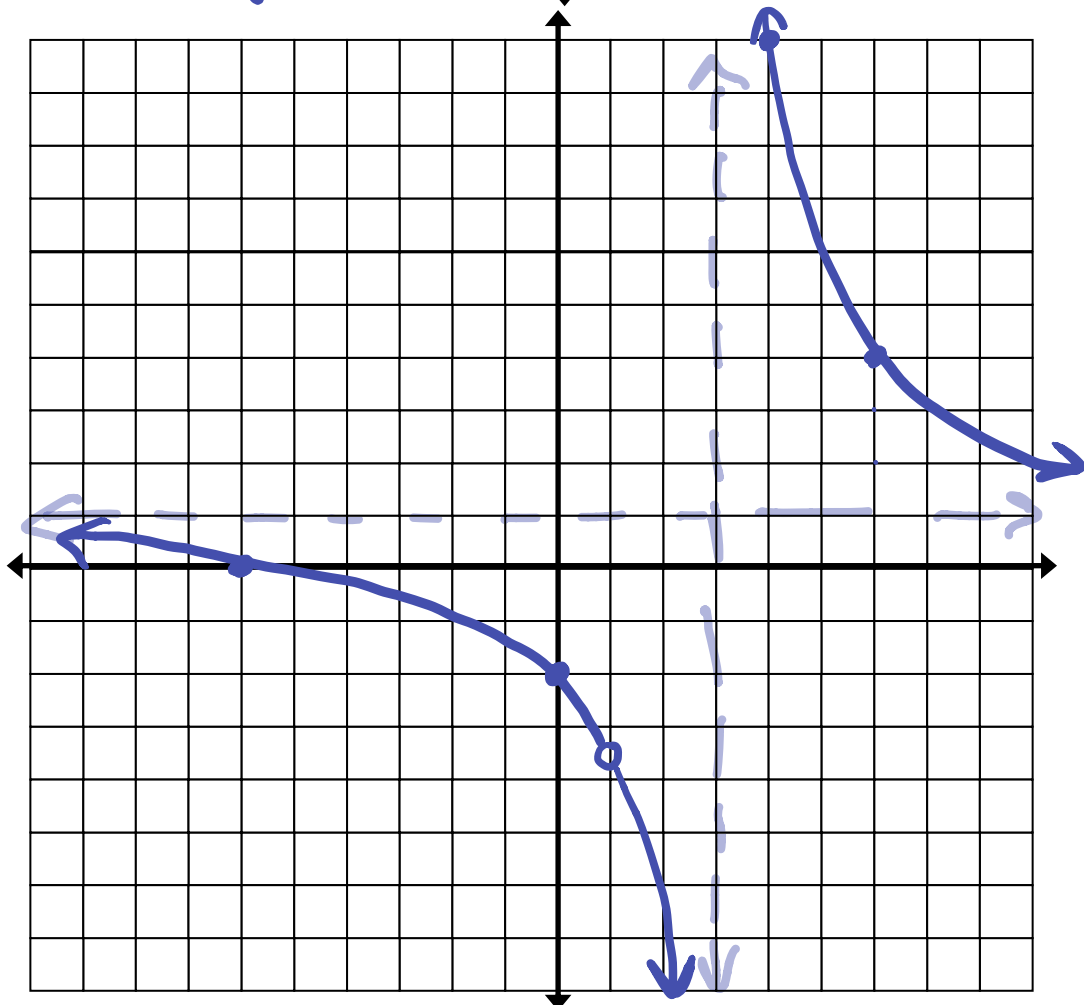
HA: $y = 0$

Holes: $x = -3$

$$4. y = \frac{x^2 + 5x - 6}{x^2 - 4x + 3}$$

$\frac{(x+6)(x-1)}{(x-3)(x-1)}$

$$y = \frac{x+6}{x-3}$$



Asymptotes:

Vert: $x = 3$

HA: $y = 1$

Holes: $x = 1$